

# *An Inverse Method to Estimate the Principal Thermal Conductivities of Composite Material*

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**Abstract—** This paper reports methodology for simultaneous estimation of principal thermal conductivity of a composite made of aluminium core with aluminium face sheet bonded by using an adhesive layer, using inverse method. The direct problem consists of a three dimensional heat conduction equation in an orthotropic composite medium. The direct problem is solved by using fluent to estimate the temperature distribution on the test sample by varying the thermal conductivity at steady state condition. An artificial neural network is used as an inverse tool to retrieve the thermal conductivity. Sixty sample data temperature values are used to train a neural network by randomly varying the thermal conductivity within a range of  $0.5 \leq k_x, k_y \leq 7$  W/mK and  $0.1 \leq k_z \leq 3$  W/mK. A detailed neuron independence study has been carried out to find the number of neurons required to train the network based on the mean square error and regression values. The methodology is demonstrated by retrieving thermal conductivity from the trained network by giving a temperature data that were not used for the training. The retrieved values of thermal conductivities from the trained network were found to deviate to a maximum of  $\pm 15\%$  from the actual values.

**Keywords—** Composite material, Inverse heat transfer, Artificial neural network.

## I. INTRODUCTION

A composite material is one which is composed of at least two constituents or elements adhered together to produce material properties that are different from the elemental properties and are essentially insoluble in each other. Composite materials are most often an innovative technological solution to improve and create more competitive products in many industries. Their properties offer numerous advantages compared to those of traditional materials. Composite materials find extensive use in many engineering applications because of their well-known superior thermo physical properties. Knowledge of thermal conductivity of the composite play an important role in thermal designing.

Sawaf et al. [1] proposed an inverse analysis to estimate linearly temperature dependent thermal conductivity components and specific heat capacity for an orthotropic solid. Dilek et al. [2] investigated numerically and experimentally the effective thermal conductivity of particle filled polymer composite. In numerical study, the finite element program ANSYS is used to calculate the thermal conductivity of the composite. A modified hot wire method is used to measure the thermal conductivity of the composite. Sun et al. [3] developed an inverse parameter estimation technique to determine the temperature dependent thermo physical

properties of anisotropic materials. Deng and Hwang [4] applied neural network (ANN) to compute the temperature distribution in forward heat conduction problems and solved inverse heat conduction problems by using back propagation neural network (BPN) to identify the unknown boundary conditions. Thomas et al. [5] developed an experimental procedure for the simultaneous estimation of the thermal conductivity tensor and specific heat of orthotropic composite material, which uses an ordinary least square method for inverse methodology. Rajlakshmi et al. [6] carried out a computational and experimental investigation on thermal conductivity of particle reinforced epoxy composite for various filler concentration. Chanda et al. [7] experimentally estimated the thermal conductivities of an anisotropic composite medium using inverse heat transfer.

In the preceding literature various methods for the estimation of thermal conductivity of composite materials which includes experimental, numerical and existing empirical relations have been reviewed. Some of the research papers discuss about the simultaneous estimation of thermal conductivity. Most of the papers deals with the estimation of effective thermal conductivity of the composite materials, but the accuracy of such has to be strengthened by conducting experiments in composite materials. Thermal conductivity of composite material vary along each directions, hence the values of  $k_x, k_y$  and  $k_z$  play an important role in designing. Among the various techniques used for estimation of thermal conductivity of the composite material inverse heat conduction problem using artificial neural network is scarce. The objective of the present study is to propose a methodology to simultaneously estimate the principal thermal conductivities of the composite material using inverse method.

## II. METHODOLOGY

The methodology is defined in two steps, (1) Forward model and (2) Inverse model. Forward model is used to determine the temperature distribution on the composite material using Fluent. Inverse model is used to retrieve the thermal conductivity of the composite material using artificial neural network (ANN). The orthotropic material consider here is a honey comb composite material made with aluminium core (Al-5056) and aluminium face sheet bonded using an adhesive layer used in spacecraft application The thermal conductivity range of the material is ( $0.5 \leq k_x, k_y \leq 7$  W/mK and  $0.1 \leq k_z \leq 3$  W/mK) reported by Thomas et al.[5].

The geometry under consideration is a square plate of dimensions 'a' along the x direction and 'b' along the y direction (a=b). The thickness of the sample is 'c' along the z direction. The schematic of the problem geometry is shown in Fig-1 having a cold plate (180x180mmx15mm) made of aluminium used as conductive sink. The test sample (180x180mmx5mm) reject heat to the sink via a copper ring. A square copper ring (180x180x5mm with square cavity of 140x140mmx5mm) is placed between test sample and cold plate. A heater (30x30mmx1mm) is placed on the top of the test sample. The boundary conditions applicable to the problem are shown in Table-1. The copper ring alters the path of conductive heat rejection from the test sample to the cold plate by restricting it to pass through itself, thereby helping to create a much larger in plane temperature gradient on the test sample.

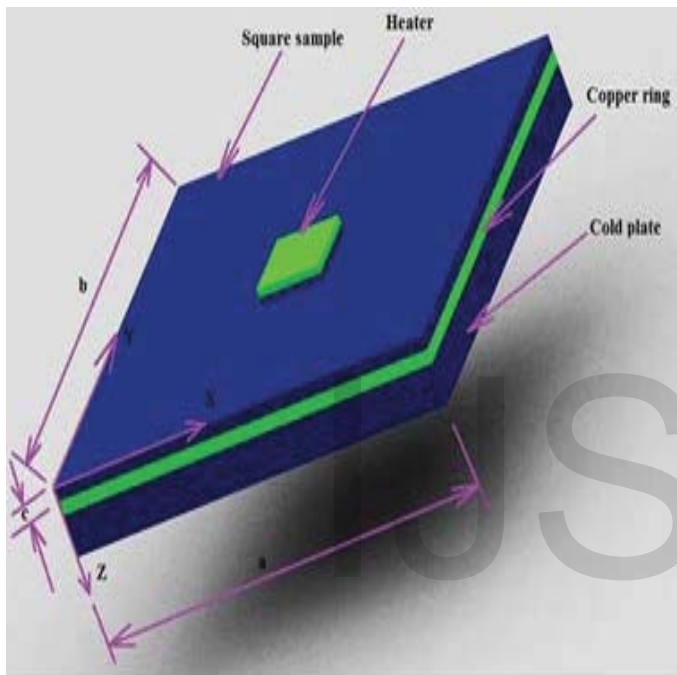


Fig. 1. Isometric view of the problem geometry

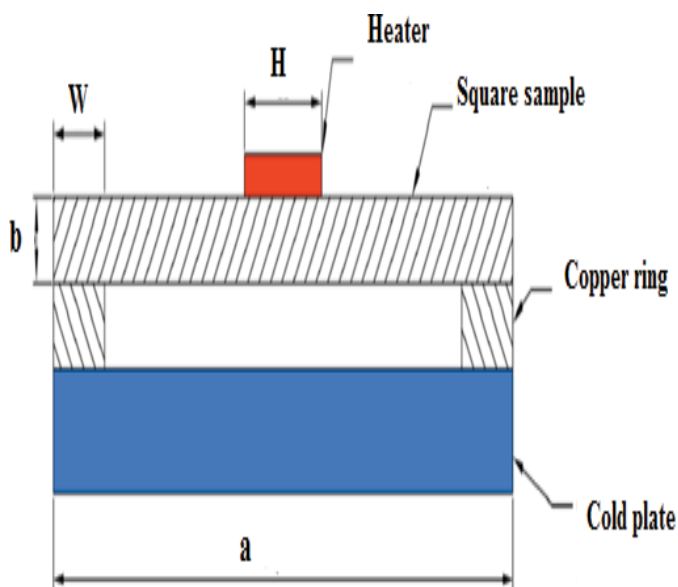


Fig. 2. Schematic of the problem in xz plane

TABLE 1. BOUNDARY CONDITIONS

| Boundary condition                  | Valid for  |
|-------------------------------------|--|
| Heat input                          | Heater   |
| $\frac{\partial T}{\partial z} = 0$ | $0 \leq x \leq \frac{a-H}{2}, 0 \leq y \leq b, z=0$<br>$\frac{a-H}{2} \leq x \leq \frac{a+H}{2}, 0 \leq y \leq \frac{b-H}{2}$ and<br>$\frac{b+H}{2} \leq y \leq b, z=0$<br>$\frac{a+H}{2} \leq x \leq a, 0 \leq y \leq b, z=0$ |
| $\frac{\partial T}{\partial y} = 0$ | $0 \leq x \leq a, 0 \leq z \leq c, y=0;$<br>$0 \leq x \leq a, 0 \leq z \leq c, y=b$  |
| $\frac{\partial T}{\partial x} = 0$ | $0 \leq y \leq b, 0 \leq z \leq c, x=0;$<br>$0 \leq y \leq b, 0 \leq z \leq c, x=a$  |
| $\epsilon=0.06$                     | Copper ring all inner surfaces.  |
| Cold plate                          | Isothermal temperature = 213K  |

The governing equations on the specimens are given below

1) Heater 
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{k} = 0 \quad (1)$$

2) Test sample 
$$\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) = 0 \quad (2)$$

3) Fluid Momentum equation

X- Momentum equation;

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial P}{\partial x} \quad (3)$$

Y- Momentum equation;

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial P}{\partial y} \quad (4)$$

Z- Momentum equation;

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial P}{\partial z} - g\beta\Delta T \quad (5)$$

In this study Boussinesq approximation is incorporate with the simulation to know the effect of convection heat transfer. The temperature dependent density variation of the material is known as the boussinesq approximation. The geometry of the problem has modelled in gambit 2.4.6 and the simulation has done by using fluent 6.3.26.

A. Artificial neural network

An artificial neural network (ANN), usually called neural network (NN), is a mathematical model or computational model that is inspired by the structure and/or functional aspects of biological neural network. A neural network consists of an interconnected group of artificial neurons, and it processes information using a connectionist approach to computation. The objective of the neural network is to transform the inputs into meaningful outputs. In the present work artificial neural network is used as an inverse tool to retrieve the thermal conductivity value. Fig-3 shows the schematic diagram of artificial neural network. The schematic diagram of the methodology is shown in Fig-4.

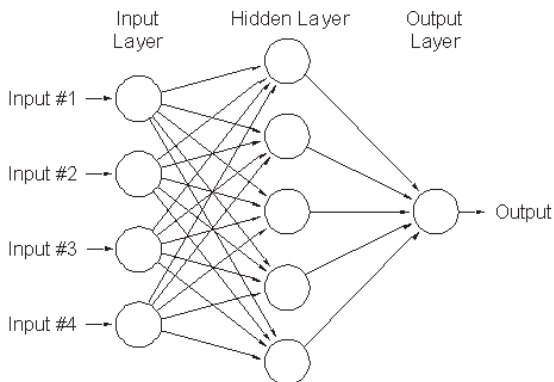


Fig. 3. Schematic diagram of ANN

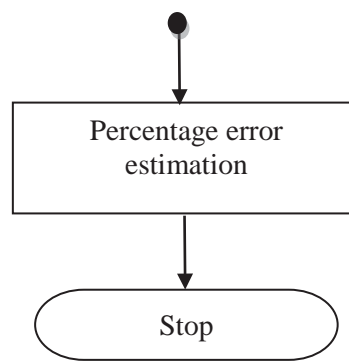


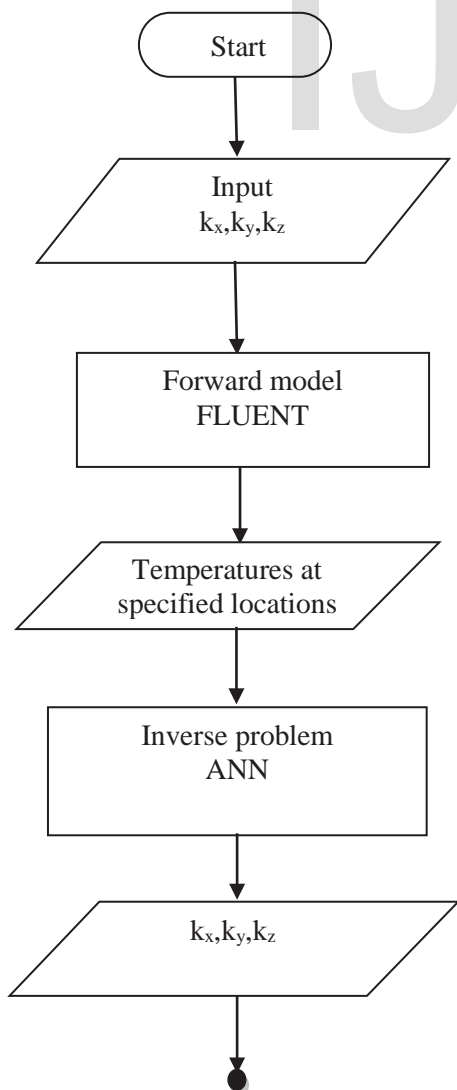
Fig. 4. Schematic diagram of the methodology

III. RESULT AND DISCUSSION

This present work proposed a methodology to estimate the orthotropic thermal conductivity of the composite material using inverse heat transfer analysis. The direct problem was solved by simulation done by using Fluent 6.3.26 and the inverse problem was solved by using artificial neural network. The sample consist of an aluminium core and aluminium face sheet bonded using an adhesive layer is selected for the present work.

Neural network training is the first step in thermal conductivity measurement of composite material by using inverse method. In the present work an artificial neural network is used as an inverse tool and it is prepared using standard MATLAB modules. To train a neural network temperature data is used as the input and thermal conductivity as the output. The temperature data for the network is obtained from the forward model. The forward model used simulation by using fluent software to estimate the temperature distribution at specified locations on the test sample. By keeping the value of heat input as constant i.e.,  $q=2.5$  W and varying the thermal conductivities of the composite ( $0.5 \leq k_x, k_y \leq 7$  W/mK and  $0.1 \leq k_z \leq 3$  W/mK) the temperature distributions at twenty two specified locations are measured. Fig-5 shows the contour of the static temperature in Kelvin corresponds to  $q=2.5$  W,  $k_x=4.38$  W/mK,  $k_y=4.11$ W/mK and  $k_z=0.68$  W/mK. Sixty samples of temperature data are estimated by randomly varying the thermal conductivity values.

Sixty samples are chosen for train a neural network. Out of this sixty samples 42 are used for training, nine samples are used for validation and the remaining nine is used for testing. A neuron independence study has been carried out to fix the number of neurons require for the training based on mean square error and regression. The number of neurons is varied from 1 to 13 to perform the neuron independence study. Based on neuron independence study the no. of hidden neurons required for the training is fixed as 9. The details of neuron independence study is listed in Table 2.



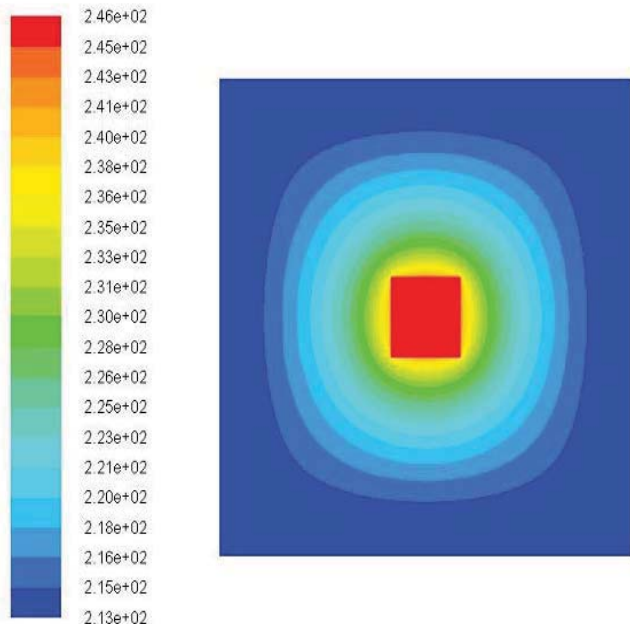


Fig. 5 contour of the static temperature in kelvin

TABLE 2. RESULT OF NEURON INDEPENDENCE STUDY

| Number of neurons | MSE            | R <sup>2</sup> |
|-------------------|----------------|----------------|
| 1                 | 1.361          | 0.7957         |
| 2                 | 0.2085         | 0.9746         |
| 3                 | 0.1934         | 0.9732         |
| 4                 | 0.0043         | 0.9994         |
| 5                 | 0.0021         | 0.9997         |
| 6                 | 0.0019         | 0.9998         |
| 7                 | 0.0289         | 0.9961         |
| 8                 | 0.00044        | 0.9999         |
| <b>9</b>          | <b>0.00019</b> | <b>0.9997</b>  |
| 10                | 0.01015        | 0.9987         |
| 11                | 0.0397         | 0.995          |
| 12                | 0.0344         | 0.995          |
| 13                | 0.0372         | 0.994          |

After training, testing and validating the ANN network is used as the inverse tool to obtain the thermal conductivity for a given temperature distribution. Real experiments can be conducted with the given heat input for unknown thermal conductivity within the prescribed range. There are eight temperature data values are used as the input to the trained network which are not used for the trained network to retrieve the values of thermal conductivity. Table-2 shows the actual and retrieved values of thermal conductivity. From the table it clears that the percentage deviation of the predicted values from the actual values are within  $\pm 15\%$ . The actual and the predicted values of the thermal conductivity of the composite material is shown in Fig-6 and the predicted values are found to be deviate within  $\pm 15\%$  from the actual value.

CONDUCTIVITY

| Forward model<br>(k <sub>actual</sub> ) |                |                | Inverse model<br>(k <sub>predicted</sub> ) |                |                |
|---|----------------|----------------|--|----------------|----------------|
| k <sub>x</sub>                          | k <sub>y</sub> | k <sub>z</sub> | k <sub>x</sub>                             | k <sub>y</sub> | k <sub>z</sub> |
| 4.38                                    | 4.11           | 0.68           | 4.42                                       | 4.18           | 0.74           |
| 3.5                                     | 1.45           | 1.7            | 3.51                                       | 1.46           | 1.53           |
| 6.18                                    | 2.93           | 2.55           | 6.13                                       | 2.95           | 2.38           |
| 5.24                                    | 4.75           | 1.57           | 5.24                                       | 4.76           | 1.71           |
| 0.73                                    | 2.37           | 1.98           | 0.67                                       | 2.37           | 2              |
| 1.75                                    | 0.8            | 0.9            | 1.7  | 0.76           | 0.94           |
| 2.43                                    | 0.9            | 0.3            | 2.32                                       | 0.98           | 0.29           |
| 5.6                                     | 5.32           | 1.1            | 5.68                                       | 5.24           | 1.2            |

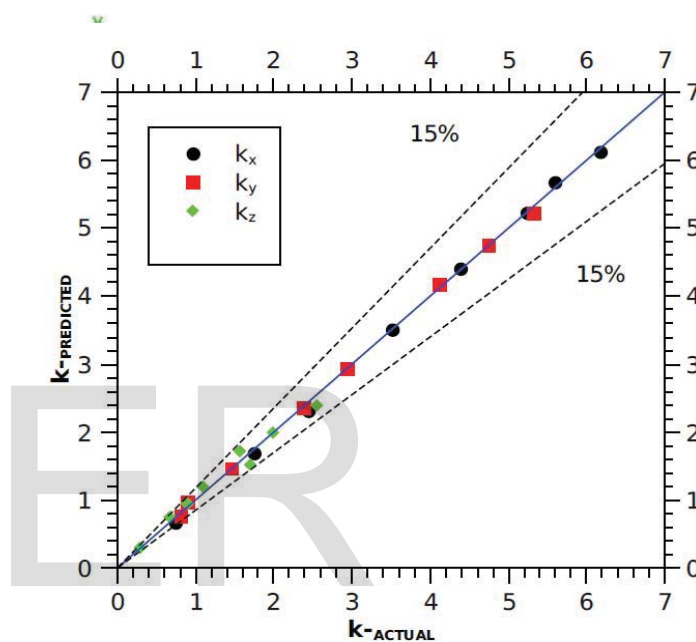


Fig. 6. Parity plot showing the variation of retrieved values from the actual value

The temperature data from fluent analysis can be replaced by actual measured temperature from experiments to retrieve the value of thermal conductivity. This is the practical application of such a methodology.

IV. CONCLUSION

The present work proposed a methodology to estimate the orthotropic thermal conductivity of anisotropic honey comb material made by aluminium core and aluminium face sheet used for satellite application using an inverse analysis. The direct problem was solved by simulation that have been done by using the fluent 6.3.26 software. The inverse problem was solved using ANN. Corresponding to the a given value of k<sub>x</sub>, k<sub>y</sub> and k<sub>z</sub> the temperature distribution is calculated using fluent (which can also replaces by experiments) and the temperature data thus obtained are used as input to the neural network. Sixty samples are used to train the network, a detailed neuron independence study has been carried out to find out the number of neurons require for train the network. Based on the values of mean square error and regression the number of neurons required for the network has obtained and it is fixed as nine. There are eight samples temperature values are given as the input to the trained network which are not used for the training to retrieve the thermal conductivity



values. The values retrieved from the trained network are found to deviate within  $\pm 15\%$  from the actual value.

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